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Analysis of Probabilistic Flooding in VANETs

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It is known that probabilistic flooding in mobile ad hoc networks is characterized by phase transition phenomena [1], similar to the ones observed in the context of random graphs and percolation theory [2], which suggest the existence of a critical rebroadcast probability value beyond which high reachability is achieved with high probability. The phenomenon has been recently observed in the context of Vehicular Ad Hoc Netwoks [3] which constitute a subclass of MANETs with, however, special characteristics such as the confined network topology. It is thus important to verify the existence of such phase transition phenomena anlytically in the context of VANETs. In this technical report we first utilize a simple mathematical model of a vehicular network on a single lane road to derive a difference equation which can be used to find numerical values of the probability of all vehicles receiving the critical message as a function of the retransmission probability. The solution of this difference equation is shown to exhibit phase transition phenomena similar to the ones observed using simulations. We then extend our analysis to a two lane road and we derive a lower bound on the probabability of all vehicles receiving the message. It is worth noting that the analysis reveals the inherent existence of recursion in the analysis of probabilistic flooding schemes.

A. *One Lane Analysis*

We assume a straight line roadway section on which n equidistant vehicles move along a straight line. The vehicles have a common transmission range and the distance between the vehicles is set to half the transmission range. The latter implies that when a vehicle transmits a critical message, four vehicles in the vicinity of the transmitter can receive the message, two in front and two at the back. The vehicles are indexed by integers $\{1, 2, 3, ..., n\}$ in ascending order from left to right, with vehicle 1, denoted by v_1 being the left most vehicle. Without loss of

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generality we also assume that vehicle 1 is the initiator node which generates the critical message and broadcasts it for the first time. The rest of the vehicles employ probabilistic flooding in order to disseminate the message to all vehicles. So, a vehicle, upon receiving the critical message for the first time decides to rebroadcast the message with probability p and decides not to rebroadcast the message with probability $1-p$. The above setting can be represented by a graph $G(V,E)$ where V is the set of nodes and E is the set of edges. Each vehicle v_i is considered a node n_i in this representation and two nodes n_i n_j are associated with an edge $(i,j) \in E$ if the one lies in the transmission range of the other. Since the transmission range of the vehicles is equal to double the common distance between the vehicles, the graph can be defined as follows:

$$G(V,E), \qquad V = \{n_i, i\epsilon[n]\} \tag{1}$$

$$E = \{(i,i+1),(i,i+2)|1 \leq i \leq n-2\} \tag{2}$$

Assume now that in the above setting the initiator node broadcasts the initial message and that the rest of the nodes employ probabilistic flooding to disseminate the message to all nodes. We represent the nodes which have sent or received a critical message by a graph $G_b = (V_b, E_b)$ where V_b is the set of nodes which have sent or received a critical message. An edge (i,j) lies in E_b if node n_i has sent a message to n_j or if node n_i has received a message from n_j. Note that $V_b \subseteq V$, $E_b \subseteq E$. If the critical message has been received by all vehicles then $V_b = V$. The following transforms the reachability problem into a graph theoretic connectivity problem:

Lemma 1: V_b is a connected graph

Proof: Assume in contradiction that G_b consists of more than one non-empty connected disjoint components G_1, G_2, G_3, G_i, $i \leq n$. Denote by G_1 the component which contains the initiator node n_1. The initiator node n_1 lies in G_b for sure since it is the one which initially broadcasts the message. Since $G_j, j = \{2,3,..,i\}$ are disconnected from G_1, it means that no node from G_1 was able to transmit the unknown message to any node in $G_j, j = \{2,3,..,i\}$. Since $G_j, j = \{2,3,..,i\}$ can only receive the critical message from G_1, it implies that $G_j, j = \{2,3,..,i\}$ is empty. Since we initially assumed $G_j, j = \{2,3,..,i\}$ to be non-empty we have reached a contradiction. ■

Our objective is to compute the probability of all vehicles receiving the critical message. That is we aim at calculating the probability of $V_b = V$. We denote this event by U and we denote the probability of U occurring as $\Pi(n)$, since $|V| = n$. To compute $\Pi(n)$ we consider the application of the probabilistic flooding algorithm on the considered topology as the experiment and we represent the probability space of this experiment using a probability tree diagram. The tree corresponding to n nodes is denoted by $T(n)$. The tree diagram in the case of $n = 5$ is shown in Fig. 1. In this representation, each node B_i denotes the event that node i rebroadcasts the message assuming that it has received it, whereas NB_i denotes the event that node i does not rebroadcast the message. The probability that node i rebroadcasts the message is equal to p whereas the probability that node i does not rebroadcast the message is equal to $1-p$. This dictates the branch preceding each node B_i and NB_i. To each branch preceding node B_i we associate a probability p whereas to each branch preceding NB_i we associate a probability $1-p$. Each of the possible outcomes at the $n'th$ level of the tree is denoted by S and is indexed by $k \in K$. Note that $U = \bigcup_{k \in K} S_k$ is the event of all vehicles receiving the message which is the event of success.

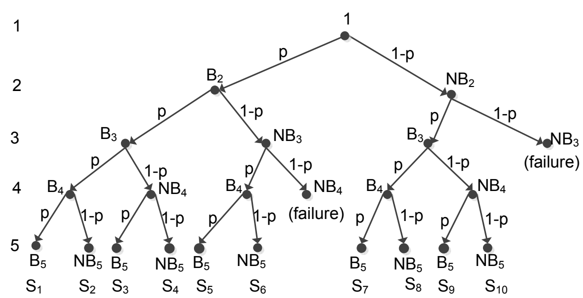

Fig. 1: Probability tree diagram for the case of five nodes.

In T(5) note that when two consecutive nodes $i, i+1, i \leq 3$ do not broadcast then the algorithm terminates without success. On the other hand if node i broadcasts then the algorithm continues and we may consider node i as the initiator of the tree rooted on i denoted by $T(n-i+1)$. Finally, if node i did not broadcast, the only case that allows further propagation of the critical message, is node $i+1$ to broadcast the message. In this case, we may consider node $i+1$ an initiator of a tree rooted at $i+1$, The latter tree is denoted by $T(n-i)$. We can now redraw, the tree diagram for the case of n nodes taking into account internal trees of similar structures. This leads to recursion. The tree is shown in Fig. 2.

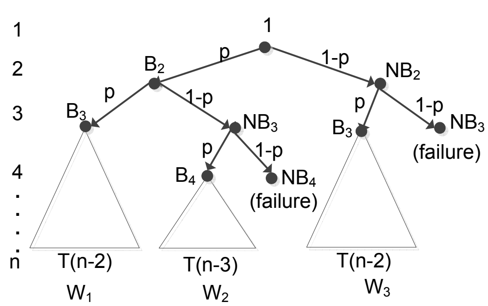

Fig. 2: Probability tree diagram indicating recursion.

The nodes B_3 at level 3 of the tree are the roots of trees $T(n-2)$. The node B_4 at level 4 of the tree is the root of tree $T(n-3)$. The set of successful outcomes generated by each of the trees are denoted by W_1, W_2 and W_3 respectively as shown in the diagram. From the tree diagram one can derive the following difference equation.

$$
\begin{aligned}
\Pi(n) &= P(W_1) + P(W_2) + P(W_3) \\
&= pp\Pi(n-2) + p(1-p)p\Pi(n-3) + (1-p)p\Pi(n-2) \\
&p^2\Pi(n-2) + p^2(1-p)\Pi(n-3) + (1-p)p\Pi(n-2) \\
&p\Pi(n-2) + p^2(1-p)\Pi(n-3)
\end{aligned}
\tag{3}
$$

The initial conditions of this difference equation are obviously $\Pi(1) = \Pi(2) = \Pi(3) = 1$. The solution of this difference equation yields the probability of all vehicles receiving the critical message in the setting described above. The difference equation is not trivial to solve in closed form, so in order to gain insights on its behavior we solve it numerically and in Fig. 3 we plot $\Pi(n)$ as a function of p for different values of n. We observe strictly increasing functions of p. In addition, we observe areas where the probability of all vehicles receiving the message is low and areas where the probability is high. The transition from one area to the other becomes more and more abrupt as the number of nodes increases. The above analysis verifies the existence of phase transition phenomena associated with probabilistic flooding despite the simplicity of the considered model.

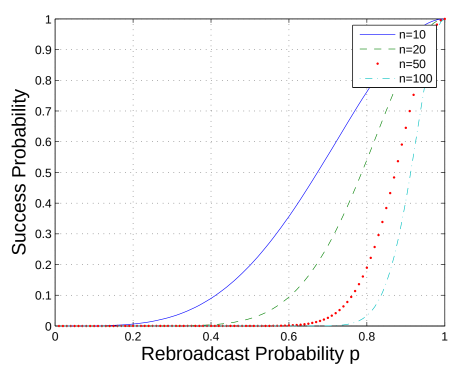

Fig. 3: Probability of all vehicles receiving the critical message versus the rebroadcast probability for different values of n.

B. Two Lane Analysis

We now assume 2n vehicles on a two lane straight line roadway section as shown in Fig 4. The vehicles are equidistant and have a common transmission range. The vehicles in the upper lane are indexed by integers {1, 2, 3, ..., n} in ascending order from left to right, with vehicle 1, denoted by 1U being the left most vehicle and vehicles in the lower lane are also indexed by integers {1, 2, 3, ..., n} in ascending order from left to right, with vehicle 1, denoted by 1D being the left most vehicle. The distance between the vehicles is set to half the transmission range. This implies that when a node iU (iD) broadcasts a message, the message can be received by

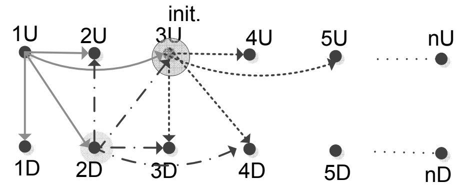

Fig. 4: The two lane graph L_{2n}. The nodes receiving the critical message when the initiator node and an arbitrary intermediate node broadcast are illustrated.

nodes $iU-2, iU-1, iU+1, iU+2$ of the upper lane of the graph ($iD-2, iD-1, iD+1, iD+2$ of the lower lane) and by nodes $iD-1, iD, iD+1$ of the lower lane of the graph ($iU-1, iU, iU+1$ of the upper lane of the graph). Without loss of generality we assume that vehicle 1U is the initiator node which generates the critical message and broadcasts it for the first time. The rest of the vehicles employ probabilistic flooding in order to disseminate the message to all vehicles. So, a vehicle, upon receiving the critical message for the first time decides to rebroadcast the message with message with probability p and decides not to rebroadcast the message with probability 1-p. Such a two lanes graph of n nodes on each lane is denoted by L_{2n}. The L_{2n} graph together with the broadcast range of an illustrative number of vehicles is shown in Figure 4.

We wish to compute the probability of all nodes in the L_{2n} graph receiving the message. This is denoted by $\mathsf{P}(\mathsf{Success}\, L_{2n})$. In general, we denote by P(X), the probability of event X to occur, where X is any event in the sample space of the probabilistic flooding experiment we are considering. We denote, for any node iU (iD, respectively) the event that iU receives and broadcasts the message, as iUB (iDB). In addition, the event that iU (iD) does not broadcast the message is denoted as iUNB (iDNB). For clarity of presentation in the subsequent discussion we assume that all the definitions that apply for vehicles in the upper lane also apply for the vehicles in the lower lane with the U notation replaced by D.

Lemma 2: Assume application of the probabilistic algorithm on L_{2n}. When nodes $2\mathrm{U}, 3\mathrm{U}, 1\mathrm{D}$ and $2\mathrm{D}$ do not broadcast the message after receiving it, the algorithm terminates and nodes $4\mathrm{U}$ and $3\mathrm{D}$ do not receive the message. The latter nodes are the first nodes that may not receive the message.

Proof: Nodes $2\mathrm{U}, 3\mathrm{U}, 1\mathrm{D}$ and $2\mathrm{D}$ receive the message due to the initial broadcast by the initiator. Nodes $4\mathrm{U}$ and $3\mathrm{D}$ may receive the message only from nodes $2\mathrm{U}, 3\mathrm{U}, 1\mathrm{D}$ and $2\mathrm{D}$. If none of them broadcasts, the message will never be received by nodes $4\mathrm{U}$ and $3\mathrm{D}$. Thus, the algorithm terminates. ■

So, by Lemma 2, in order for the algorithm to have positive probability to succeed on L_{2n}, it must be that the algorithm does not terminate at nodes $4\mathrm{U}$ and $3\mathrm{D}$. The complement of the event of both $4\mathrm{U}$ and $3\mathrm{D}$ not receiving the message consists of the following disjoint events.

- E_1= 3UB&2DB

- E_2=3UB&2DNB
- E_3=3UNB&2DB
- E_4=3UNB&2DNB and the message is received by at least one of the nodes 4U, 3D from at least one of the nodes preceding nodes 3U, 2D, i.e. at least one of the nodes 1D,2U broadcast it.

The above events are disjoint since each of them contains the complement of a subset of any of the other events. Denote as $\mathsf{P}(\mathsf{Success}\, L_{2n}\&E_i)$ the probability of all nodes in L_{2n} receiving the message when event E_i has occurred, $i \in [4]$. Since the events E_i are disjoint it follows that

$$\mathsf{P}(\mathsf{Success}\, L_{2n}) = \sum_{i=1}^{4} \mathsf{P}(\mathsf{Success}\, L_{2n}\&E_i) \tag{4}$$

Before computing the probability of each of the events $\{E_1, \cdots, E_4\}$ to happen, we consider a partition of E_4 which represents all the possible ways with which the event can occur. The partition is the following:

$$E_4 = \bigcup_{i=1}^{3} E_{4/i} \tag{5}$$

where events $E_{4/i}$, $i = \{1, 2, 3\}$ are given by:

- ($E_{4/1}$)=2UB&1DNB&3UNB&2DNB
- ($E_{4/2}$)=2UB&1DB&&3UNB&2DNB
- ($E_{4/3}$)=2UNB&1DB&3UNB&3DNB

The reasoning behind this partition is that the nodes that precede nodes 3U and 2D are nodes 2U and 1D. So, in order to partition event E_4, one has to consider all possible actions of nodes 2U and 1D. Excluding the event of both nodes not broadcasting which is not allowed by the definition of E_4, the rest of the combinations are 2UB&1DNB, 2UB&1DB, 2UNB&1DB. When one combines the latter cases with 3UNB&2DNB which also stems from the definition of E_4 one obtains the above events. From equation (4) and the fact that $E4/i$ are a partition of E_4 it follows that :

$$\mathsf{P}(\mathsf{Success}\, L_{2n}) = \sum_{i=1}^{3} \mathsf{P}(\mathsf{Success}\, L_{2n}\&E_i) + \sum_{i=1}^{3} \mathsf{P}(\mathsf{Success}\, L_{2n}\&E_{4/i}) \tag{6}$$

1) Computation of Probability of Success on Event E_1: We first compute the probability of success of the algorithm when event E_1 occurs, i.e. we compute $\mathsf{P}(\mathsf{Success}\, L_{2n}\&E_1) = \mathsf{P}(\mathsf{Success}\, L_{2n}\&(\text{3UB\&2DB}))$. Event E_1 is illustrated in Figure 5.

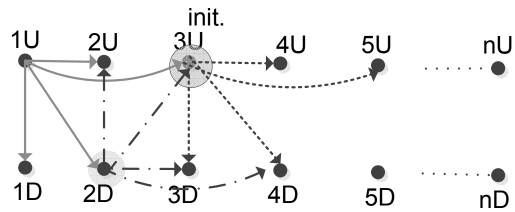

Fig. 5: The event $E_1 = \text{3UB\&2DB}$. Nodes 3U and 2D broadcast the message. Dotted arrows indicate receiving nodes due to 3U broadcasting whereas line dotted arrows indicate receiving nodes due to 2D broadcasting.

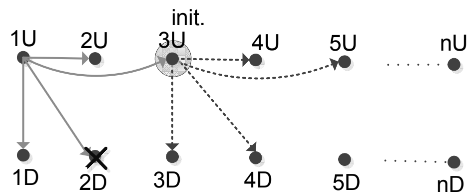

Fig. 6: The event $E_2 = \text{3UB\&2DNB}$. Node 3U broadcasts the message. Dotted arrows indicate receiving nodes due to 3U broadcasting.

Note that $\mathsf{P}(\text{3UB\&2DB}) = p \cdot p = p^2$, since each of the nodes 2D and 3U receive the message from the initiator node and broadcast with probability p, independent of each other. We observe that when 3U and 2D broadcast the message, they *act as initiators* of two lane graphs. When 3U broadcasts the message it acts as the initiator of a two lane graph consisting of $2(n-2)$ nodes whereas when 2D broadcasts the message it acts as the initiator of a two lane graph consisting of $2(n-1)$ nodes. The receiving nodes when 2D broadcasts is a subset of the receiving nodes when 3U broadcasts. We thus continue the analysis assuming that only 3U broadcasts. This assumption creates a lower bound on the calculated success probability but due to the preceding observation we expect this lower bound not to be conservative. It follows that:

$$\mathsf{P}(\mathsf{Success}\, L_{2n}\&E_1) \geq p^2 \cdot \mathsf{P}(\mathsf{Success}\, L_{2(n-2)}) \tag{7}$$

2) Computation of Probability of Success on Event E_2: We now compute $\mathsf{P}(\mathsf{Success}\, L_{2n}\&E_2) = \mathsf{P}(\mathsf{Success}\, L_{2n}\&(\text{3UB\&2DNB}))$. Event E_2 is illustrated in Figure 6.

Note that $\mathsf{P}(\text{3UB\&2DNB}) = p \cdot (1-p)$. Nodes 3U and 2D receive the message from the initiator node and the former broadcasts with probability p and the latter does not broadcast with probability $1-p$, independently of each other. We observe that node 3U acts as the initiator of a two line graph consisting of $2(n-2)$ nodes and it thus follows:

$$\mathsf{P}(\mathsf{Success}\, L_{2n}\&E_2) = p \cdot (1-p) \cdot \mathsf{P}(\mathsf{Success}\, L_{2(n-2)}) \tag{8}$$

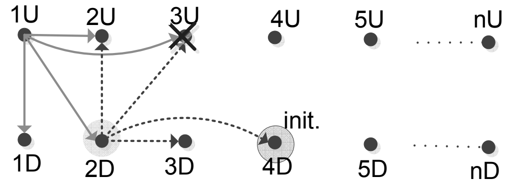

Fig. 7: The event $E_3 = 3\text{UNB}\&2\text{DB}$. The broadcasting nodes are cycled. 4D is considered to be the initiator node in order to calculate a lower bound on the success probability.

3) Computation of Probability of Success on Event E_3: We now compute $\mathsf{P}(\text{Success } L_{2n}\&E_3) = \mathsf{P}(\text{Success } L_{2n}\&(3\text{UNB}\&2\text{DB}))$. Event E_3 is illustrated in Figure 7.

Note that $\mathsf{P}(3\text{UNB}\&2\text{DB}) = (1-p)\cdot p$ since nodes 2D and 3U receive the message from the initiator node and the former broadcasts with probability p and the latter does not broadcast with probability $1-p$, independently of each other. If node 2D is considered as the initiator node, 3U which receives the message from 2D does not rebroadcast since it has received the message from a previous transmission. So, the probability of success of the algorithm in this case, is less than the probability of success of a two lane graph of $2(n-1)$ nodes,generated assuming that 2D is the initiator. This upper bound on the success probability is not desirable as a lower bound is pursued. Noting that node 4D receives the message from 2D, we consider 4D to be the initiator. This choice creates a lower bound on the success probability as transmissions of previous nodes are not accounted for in the success probability of $L_{2(n-3)}$, generated when 4D is assumed to be the initiator. We calculate a lower bound on the probability of 4D becoming an initiator node by broadcasting when E_3 occurs as follows:

$$\mathsf{P}(3\text{UNB}\&2\text{DB}\&4\text{DB}) \geq p\cdot(1-p)\cdot p, \tag{9}$$

The lower bound is due to the fact 4D may receive the message not only from node 2D but also from node 3D. We thus obtain,

$$\begin{aligned}
\mathsf{P}(\text{Success } L_{2n}\&(3\text{UB}\&2\text{DNB})) &\geq \mathsf{P}(\text{Success } L_{2n}\&(3\text{UB}\&2\text{DNB}\&4\text{DB})) \\
&\geq p\cdot(1-p)\cdot p\cdot \mathsf{P}(\text{Success } L_{2(n-3)}) \\
&= p^2\cdot(1-p)\cdot \mathsf{P}(\text{Success } L_{2(n-3)})
\end{aligned}$$

It follows that

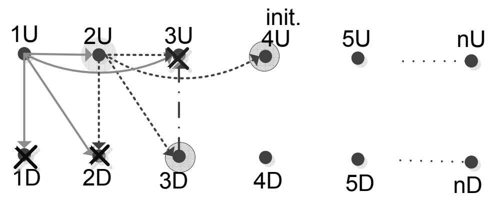

Fig. 8: The event $E_{4/1} = (\text{2UB\&1DNB})\&(\text{3UNB\&2DNB})$. The broadcasting nodes are cycled. 4D is considered to be the initiator node in order to calculate a lower bound on the success probability.

$$\mathsf{P}(\mathsf{Success}\ L_{2n}\&E_3) \geq p^2 \cdot (1-p) \cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)}) \tag{10}$$

4) Computation of Probability of Success on Event $E_{4/1}$: We now compute the probability of success of the algorithm when event $E_{4/1}$ occurs, i.e. we compute $\mathsf{P}(\mathsf{Success}\ L_{2n}\&E_{4/1}) = \mathsf{P}(\mathsf{Success}\ L_{2n}\&(\text{2UB\&1DNB})\&(\text{3UNB\&2DNB}))$. The event $E_{4/1}$ is illustrated in Figure 8.

Observe that in $E_{4/1}$, in order for the algorithm to cause all nodes to eventually receive the message, either 4U or 3D must broadcast the message. The probability to be evaluated is thus $\mathsf{P}(\mathsf{Success}\ L_{2n}\&(\text{2UB\&1DNB})\&(\text{3UNB\&2DNB})\&(\text{3DB or 4UB}))$. One may obtain a lower bound on the latter probability by neglecting any broadcast from node 3D. This leads to

$$\begin{aligned}
&\mathsf{P}(\mathsf{Success}\ L_{2n}\&(\text{2UB\&1DNB})\&(\text{3UNB\&2DNB})\&(\text{3DB or 4UB}))\\
\geq\ &\mathsf{P}(\mathsf{Success}\ L_{2n}\&(\text{2UB\&1DNB})\&(\text{3UNB\&2DNB})\&(\text{4UB}))\\
\geq\ &p \cdot (1-p) \cdot (1-p)^2 \cdot p \cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)})
\end{aligned}$$

It follows that

$$\mathsf{P}(\mathsf{Success}\ L_{2n}\&E_{41}) \geq p^2 \cdot (1-p)^3 \cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)}) \tag{11}$$

5) Computation of Probability of Success on Event $E_{4/2}$: We now compute the probability of success of the algorithm when event $E_{4/2}$ occurs, i.e. we compute $\mathsf{P}(\mathsf{Success}\ L_{2n}\&E_{4/2}) = \mathsf{P}(\mathsf{Success}\ L_{2n}\&(\text{2UB\&1DB})\&(\text{3UNB\&2DNB}))$. The event $E_{4/2}$ is illustrated in Figure 9.

Observe that in $E_{4/2}$, in order for the algorithm to cause all nodes to eventually receive the message either 4U

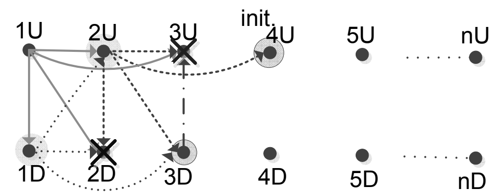

Fig. 9: The event $E_{4/2} = (\text{2UB}\&\text{1DB})\&(\text{3UNB}\&\text{2DNB})$.The broadcasting nodes are cycled. 4D is considered to be the initiator node in order to calculate a lower bound on the success probability.

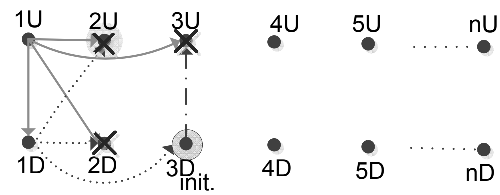

Fig. 10: The event $E_{4/3} = (\text{2UNB}\&\text{1DB})\&(\text{3UNB}\&\text{2DNB})$.The broadcasting nodes are cycled. 4D is considered to be the initiator node in order to calculate a lower bound on the success probability.

or 3D must broadcast the message. The probability to be evaluated is thus $\mathsf{P}(\text{Success } L_{2n}\&(\text{2UB}\&\text{1DB})\&(\text{3UNB}\&\text{2DNB})\&(\text{3DB or 4UB}))$. One may obtain a lower bound on the latter probability by neglecting any broadcast from node 3D. This leads to

$$\begin{aligned} & \mathsf{P}(\text{Success } L_{2n}\&(\text{2UB}\&\text{1DB})\&(\text{3UNB}\&\text{2DNB})\&(\text{3DB or 4UB})) \\ \geq \; & \mathsf{P}(\text{Success } L_{2n}\&(\text{2UB}\&\text{1DB})\&(\text{3UNB}\&\text{2DNB})\&(\text{4UB})) \\ \geq \; & p \cdot p \cdot (1-p)^2 \cdot p \cdot \mathsf{P}(\text{Success } L_{2(n-3)}) \end{aligned}$$

It follows that

$$\mathsf{P}(\text{Success } L_{2n}\&E_{4/2}) \geq p^3 \cdot (1-p)^2 \cdot \mathsf{P}(\text{Success } L_{2(n-3)}) \tag{12}$$

6) Computation of Probability of Success on Event $E_{4/3}$: We now compute the probability of success of the algorithm when event $E_{4/3}$ occurs, i.e. we compute $\mathsf{P}(\text{Success } L_{2n}\&E_{4/3}) = \mathsf{P}(\text{Success } L_{2n}\&(\text{2UNB}\&\text{1DB})\&(\text{3UNB}\&\text{2DNB}))$. The event $E_{4/3}$ is illustrated in Figure 10.

We observe that 3D may be used as an initiator to obtain the success probability when $E_{4/3}$ occurs. However, such a choice would lead to an upper bound on the probability which is undesirable. An upper bound is due to the fact that 3U cannot further broadcast the message. We thus consider node 4D as the initiator. This provides a lower bound on the success probability according to

$$\begin{aligned}
& \mathsf{P}(\mathsf{Success}\ L_{2n}\&(2\mathrm{UNB}\&1\mathrm{DB})\&(3\mathrm{UNB}\&2\mathrm{DNB})\&3\mathrm{DB}) \\
\geq\ & \mathsf{P}(\mathsf{Success}\ L_{2n}\&(2\mathrm{UNB}\&1\mathrm{DB})\&(3\mathrm{UNB}\&2\mathrm{DNB})\&3\mathrm{DB}\&4\mathrm{DB}) \\
\geq\ & (1-p)\cdot p\cdot(1-p)^2\cdot p\cdot p\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)})
\end{aligned}$$

It follows that

$$\mathsf{P}(\mathsf{Success}\ L_{2n}\&E_{43}) \geq p^3\cdot(1-p)^3\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)}) \tag{13}$$

Combining equation (6) with equations (7) - (13), we get

Theorem 3: The probability of success of the algorithm on a two line graph L_{2n} is lower bounded by

$$\begin{aligned}
\mathsf{P}(\mathsf{Success}\ L_{2n}) \geq\ & p\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-2)}) \\
& +\left(p^2+(p^2+p^3)\cdot(1-p)^3+p^5-2p^4\right)\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)})
\end{aligned}$$

Proof:

$$\begin{aligned}
& \mathsf{P}(\mathsf{Success}\ L_{2n}) \\
\geq\ & p^2\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-2)}) + p\cdot(1-p)\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-2)}) + p^2\cdot(1-p)\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)}) \\
+\ & p^2\cdot(1-p)^3\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)}) + p^3\cdot(1-p)^2\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)}) \\
+\ & p^3\cdot(1-p)^3\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)}) \\
=\ & \left(p^2+p(1-p)\right)\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-2)}) \\
+\ & \left(p^2(1-p)+p^2(1-p)^3+p^3(1-p)^2+p^3(1-p)^3\right)\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)}) \\
=\ & p\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-2)}) + \left(p^2+(p^2+p^3)\cdot(1-p)^3+p^5-2p^4\right)\cdot \mathsf{P}(\mathsf{Success}\ L_{2(n-3)})
\end{aligned}$$

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